ISSN 1822-7996 (PRINT), ISSN 2335-8742 (ONLINE)
TAIKOMOJI EKONOMIKA:
SISTEMINIAI TYRIMAI: 2018.12/1
https://doi.org/10.7220/AESR.2335.8742.2018.12.1.3

Andrejs JAUNZEMS

Endogenous Indicators of the Joint Intertemporal Financing and Investing Program

Van Horne and Vachowiz Jr. (2001, 2003) state that the criteria for selecting long-term investments is possibly the most complicated and the most disputable issue in financial management. The purpose of this paper is further in-depth understanding of this issue. The paper is intended as a continuation of discussion with highly respected Dr. Kruschwitz (1995, 2000) on the meanings of calculated endogenous rate of return (deu. *Endogene Kalkulationszinsfüße)* and endogenous capitalization (deu. *Endogene Kapitalwert*), covered in his relevant books *"Investitionsrechnung"* (6th and 8th eds.). It will be proved that the concept of endogenous calculated profitability offered by honourable Dr. Kruschwitz contains serious fallacy, which also remains in the latest editions of this famous book. The goal is to explain the contents of endogenous rates of return and endogenous capitalizations with the help of linear programming by means of the Stiefel (1960) systems of equation and theorem of duality.

Keywords: joint intertemporal financing and investing program, endogenous rate of return, endogenous capitalization, shadow prices, reduced costs.

Van Horne ir Vachowiz Jr. (2001, 2003) teigia, kad ilgojo laikotarpio investicijų pasirinkimo kriterijai tikriausiai yra sudėtingiausi ir labiausiai ginčytini investicijų valdymo klausimai. Šio straipsnio tikslas – nuodugniau išanalizuoti šią problemą. Straipsnis skirtas tolesnėms diskusijoms su didžiai gerbiamu dr. Kruschwitz (1995, 2000) apie apskaičiuotas endogeninės grąžos normos (vok. *Endogene Kalkulationszinsfüße*) ir endogeninės kapitalizacijos (vok. *Endogene Kapitalwert*) vertes, kurios pateikiamos jo knygose „Investitionsrechnung" (6 ir 8 leidimas). Straipsnyje siekiama įrodyti, kad į gerbiamo dr. Kruschwitz pasiūlytą pelningumo skaičiavimą įsivėlė rimta klaida, kuri pastebima ir naujausiuose šios knygos leidimuose. Taikant tiesinio programavimo vertes ir Stiefel (1960) lygčių sistemas bei dualumo teoremą, siekiama paaiškinti endogeninės grąžos normos ir endogeninės kapitalizacijos esmę.

Reikšminiai žodžiai: jungtinė tarplaikinė finansavimo ir investavimo programa, endogeninė grąžos norma, endogeninė kapitalizacija, orientacinės kainos, sąnaudų mažinimas.

JEL Classifications: C02/G11/M21.

Introduction

Appraisal and comparison of long-term financing and investing programs is probably one of the most contradictious question in financial management area (Van Horne, Vachowiz Jr., 2001). The surveys of capital budgeting techniques

Andrejs JAUNZEMS – prof. dr. Ventspils University College. Address: Inženieru iela 101, Ventspils, LV-3600; e-mail jaunzems@venta.lv

mentioned in popular financial management textbooks show that managers as a rule use such popular indicators of cash flows as internal rate of return, net present value and profitability index by attaching them different weights for decision-making (see, for example, Van Horne, Vachowiz Jr., 2003). What indicator must be preferable? Many books and papers till nowadays witness topicality of this question. Just one example: Sneps-Sneppe (2017a, 2017b, 2017c) discussed this issue in three papers – "How to measure the effectiveness of investments: criticism of the arguments of Pierre Masse" , "On the internal rate of return of IRR and the priority of investments" , "The problem of investment during the history of economic science" .

Jaunzems (2009) explained that the problem about priority of either NPV, or IRR does not exist because of substantial difference between these concepts. The most important tool for cash flow analysis is function NPV(i), i ≥ 0, as a whole. According to classical formulation of the Irving Fisher in his famous book "The Nature of Capital and Income. – New York, 1923" the indicator NPV(i) can be interpreted as fundamental value of an asset if opportunity cost of capital for potential buyer is i:

"The value of an asset is the present value of the expected returns. Value today always equals future cash flow discounted at the opportunity cost of capital."

Fundamental value of assets is the most used indicator for speculators, for example, for valuation of bonds (Bodie, Merton, 2000). The speculators buy undervalued assets in order to sell them later because they believe that market values will follow the theoretical fundamental value. However, IRR (or yield to maturity YTM) is the most used indicator for real business projects' profitability evaluation. Seo (2000) wrote that approximately 55% of businesspersons evaluate the long-run investment projects with help of IRR and only 10% – with help of NPV.

For all that is insufficient to analyse one or other investment or financing project separately. The most important thing in real business is the firm's environment analysis (known as PESTILB – political, economical, social, technological, international, legal, bio-environmental) and particularly wide investment-financing environment analysis. Intertemporal gains and losses from joint investing and financing projects have to be investigated complexly.

In order to stress the necessity of system approach in the investing and financing program analysis this paper reconsiders the original concepts of endogenous capital value and endogenous calculated profitability of the joint intertemporal investing and financing project (deu. *Endogene Kalkulationszinsfüße, Endogene Kapitalwerte der Investitions- und Finazierungsprojekte*) introduced by Dr. Lutz Kruschwitz in his relevant book "*Investitionsrechnung. 6. Auflage. – Walter de Gruyter, 1995*" (Kruschwitz, 1995, 2000). Both concepts will be carefully tested there. And it will be proved that this concept offered by honourable *Dr. Lutz Kruschwitz* contains serious fallacies which remain also in all latest editions of this famous book (for example, in Kruschwitz, 2001).

I realize my serious liability for such categorical assertion about failure in book presented under "*Walter de Gruyter*" brand and that is the reason why I bestow the authority upon the scientific journal to pass the sentence about correctness of my conclusions. I started the discussion already in the papers (Jaunzems, 2001, 2008) but I did not get any reclamations probably because the papers were in Latvian.

Materials and Methods

Let us consider the joint intertemporal investing and financing problem in case of determinated cash flows, formulated by Kruschwitz (1995, 2000, 2001). Our target is with help of simplex method to make explicit the mutual interdependences between different financial indicators which given information contains in implicit form. Such interdependences allow us to obtain interpretations of financial indicators.

The Model of Joint Investing and Financing Intertemporal Programming

Let us suppose that from investor's perspective the columns in the Table 1 are the *r* investing and *s* financing cash flows during *n* time periods (years) available for investor.

Let the $((n + 1) \times r)$ matrix INV := $(a_{i\,j})$ be denoted as investment matrix and the $((n + 1) \times s)$-matrix FIN := $(b_{i\,j})$ as financing matrix. Let us assume that the $((n + 1) \times 1)$-vector $M := (m_0\ m_1\ \dots\ m_n)^T \geq O$ is the cash flow accepted by investor as exogenous intertemporal money inflow. In the simplest case $m_0 > 0$, $m_1 = 0$, $m_2 = 0, \dots, m_n = 0$. It means that investor makes exogenous money investment m_0 only in the very beginning.

The special case is M = O. In this case, investor starts with zero sum and tries to finance investments only through borrowed money. In this case, arbitrage possibilities could arise. Bodie, Cane, Marcus, Perrakis and Ryan (1999) define arbitrage in a categorical form as a possibility to earn profit with zero investment and zero risk: „A zero-risk, zero-net investment strategy that still generates profits". Jaunzems (2012) investigated immunized quasi-arbitrage.

The main idea of the model is that investor is able to construct potentially feasible portfolio cash flow as linear combination of the cash flows of separates investing and separates financing projects in following form:

$$x_1\, INV_1 + x_2\, INV_2 + \dots x_r\, INV_r;$$
$$y_1\, FIN_1 + y_2\, FIN_2 + \dots y_s\, FIN_s,$$

where $x_1, x_2, \dots, x_r \geq 0$; $y_1, y_2, \dots, y_s \geq 0$; Vector $X = (x_1\ x_2\ \dots\ x_r)^T \in R^{r,1}$ will be referred as an investment plan; vector $Y = (y_1\ y_2\ \dots\ y_s)^T \in R^{s,1}$ will be referred as a financing plan; the pair of vectors (X, Y) will be called a financial program.

After implementation of financial program (X, Y) supported by exogenous cash flow M investor gets endogenous cash flow – vector Z(X, Y) := INV X + FIN Y + M. Liquidity of investing-financing plan means that $Z(X, Y) \geq O$, i. e. in each period of time the outflow is no less as inflow.

Let us assume that investment plan X and financing plan Y are potentially

Table 1

Investing and financing cash flows available for investor

Time	INV_1	INV_2		...	INV_r	FIN_1	FIN_2		...	FIN_s
0	$a_{0\,1}$	$a_{0\,2}$		...	$a_{0\,r}$	$b_{0\,1}$	$b_{0\,2}$		...	$b_{0\,s}$
1	$a_{1\,1}$	$a_{1\,2}$		...	$a_{1\,r}$	$b_{1\,1}$	$b_{1\,2}$		...	$b_{1\,s}$
...	...	...		...	...	...	...		...	...
N	$a_{n\,1}$	$a_{n\,2}$		...	$a_{n\,r}$	$b_{n\,1}$	$b_{n\,2}$		...	$b_{n\,s}$

feasible if they satisfied limitedness conditions: $O \le X \le X^\wedge$, $O \le Y \le Y^\wedge$, where $X^\wedge \in R^{r,1}$, $Y^\wedge \in R^{s,1}$.

So the set of available endogenous cash flows depending of exogenous investment vector M is $Z(M) := \{ Z \mid Z = INV\ X + FIN\ Y + M, O \le X \le X^\wedge, O \le Y \le Y^\wedge \}$.

The cash flow $W := Z - M = INV\ X + FIN\ Y$ can be interpreted as intertemporal cash bundle what investor gets as result of financial program's (X, Y) implementation instead the utilized cash bundle M. Thus, if investor pays M and chooses financial plan (X, Y) he gets W. Therefore, we can interpret the implementation of the financial program as intertemporal money bundle exchange and to relate this act to the consumer's behaviour theory. According to Irving Fisher investor is a person who makes decision about exchange of the private or borrowed cash flow M to another cash flow W. Investor always chooses the best financial program (X, Y) he can afford. It means: by given M he maximizes utility u(W(X, Y)) in the set of the feasible financial programs.

So, the problem solved by investor can be formulated as follows: $max\ \{u(Z - M) \mid Z = INV\ X + FIN\ Y + M, O \le X \le X^\wedge, O \le Y \le Y^\wedge \}$.

The contents of endogenous rates of return and endogenous capitalizations introduced by Dr. Lutz Kruschwitz will be explained with the help of linear programming by means of the Stiefel (1960) systems of equation and theorem of duality.

The Criteria of Quality of the Endogenous Cash Flow

Which cash flow W is the best for investor Mr. Jones?

There is the wide discussion in the literature about criteria of the quality of endogenous cash flow W. Let us mention some criteria.

(1) Let $d_0, d_1, \ldots, d_n$ be the intertemporal dividends required from investor. Usually $d_0 = 0$. In order to guarantee dividends $d_1, d_2, \ldots, d_n$ and to reach maximal outflow at the end of program in the model must be included constraints $w_1 \ge d_1$, $w_2 \ge d_2$, ... , $w_n \ge d_n$. The optimization problem is to find $max\, w_n$.

(2) Dr. Lutz Kruschwitz consider the objective – outflow cash level *niveau* =: ν. In this case in the model must be included constraints $w_1 \ge f_1 \cdot \nu, \ldots, w_n \ge f_n \cdot \nu$.

With help of coefficients $f_1, \ldots, f_n$ investor can take into account the time value of money. The task is to find *max* ν.

Remark. The term *niveau* is adopted from *niveler* (French) and means "level". The concept of level of outflow *niveau* maximization has a long history. Such approach we can find already in the works of Irving Fisher.

(3) The natural idea is to maximize net present value of endogenous cash flow W. In this case the objective is NPV(W(X, Y), i), where i is the rate of capitalization or opportunity cost of capital.

(4) The natural idea is also the maximization of internal rate of return of endogenous cash flow W. In this case the objective is IRR(W(X, Y)). In this case, the difficulties arise because the problem cannot be solved with help of simplex method.

Results and Discussion

Example of Joint Investing and Financing Program

It seems reasonable to explore results and provide discussion with help of example. Let us utilize the example of Dr. Lutz

Table 2

The (5×8)-matrix INV. Internal rates of return of the investment cash flows

Time	INV_1	INV_2	INV_3	INV_4	INV_5	INV_6	INV_7	INV_8
t = 0	0	–800	–700	–300	–100	0	0	0
t = 1	–500	80	500	700	106	–100	0	0
t = 2	–900	160	300	350	0	106	–100	0
t = 3	1250	320	–200	170	0	0	106	–100
t = 4	350	520	220	–1090	0	0	0	106
IRR =	0.0883	0.1002	0.1025	0.0931	0.0600	0.0600	0.0600	0.0600

Table 3

The (5×6)-matrix FIN. Internal rates of return of the financing cash flows

time	FIN_1	FIN_2	FIN_3	FIN_4	FIN_5	FIN_6
t = 0	1000	600	100	0	0	0
t = 1	–80	0	–110	100	0	0
t = 2	–388	0	0	–110	100	0
t = 3	–388	0	0	0	–110	100
t = 4	–388	–832	0	0	0	–110
IRR =	0.0800	0.0852	0.1000	0.1000	0.1000	0.1000

Kruschwitz (2000, p. 187) in order to compare two approaches and demonstrate the modest innovations of the author.

Assume that investing-financing environment for investor Mr. Jones contains available eight four-year investment projects and six four-years financing projects. The corresponding cash flows are shown in Table 2 and Table 3.

In the last row of Table 2 the IRR of related investment projects is shown. Last row of Table 3 contains the IRR of financing projects. Systemic approach denies naive recommendation to choose the financing project with the lowest IRR and investment project with the highest IRR.

Investment plan $X = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)^T$ characterizes the participation in the investing projects. Financing plan $Y = (y_1, y_2, y_3, y_4, y_5, y_6)^T$ characterizes the participating in the financing projects.

In the given example, it is supposed that participation in the investing projects INV_1, INV_2, INV_3, INV_4 is limited: $x_1 \leq 1$, $x_2 \leq 1$, $x_3 \leq 1$, $x_4 \leq 2$.

Participation in financing plan FIN_2 is limited: $y_2 \leq 1$.

We assume that participation in investing and financing projects is divisible, i. e. if it is necessary the investor is able to choose the parts of investing or financing cash flows. Such assumption allows us to employ the simplex method for sensitivity analysis. If it is necessary in the practice it is easy with help of integrity condition to take into account the situation when some or all investing and financing projects are not divisible and investor has to choose entire project.

In this example, it is supposed that m_0 = 500. It means that exogenous money m_0 = 500 is applied only as initial investment in moment t = 0. So $M = (500, 0, 0, 0, 0)^T$.

(I•F) Thus the set of available endogenous cash flows is $\{Z \mid Z = INV\ X + FIN\ Y + M;\ X \geq O,\ Y \geq O;\ x_1 \leq 1,\ x_2 \leq 1,\ x_3 \leq 1,\ x_4 \leq 2;\ y_2 \leq 1\}$.

The role of criteria in Lutz Kruschwitz model plays the money outflow's level niveau ν. Besides that, the investor claims by finishing of program gets back initially invested 500 dollars. As result the following constraints are added: $z_1 \geq 1{,}00 \cdot \nu$, $z_2 \geq 1{,}05 \cdot \nu$, $z_3 \geq 1{,}10 \cdot \nu$, $z_4 \geq 1{,}15 \cdot \nu + 500$.

The target is to find *max* ν. Further in text this problem will be called shortly as problem (I•F).

The Fallacy in the Dr. Lutz Kruschwitz Interpretations of Endogenous Rate of Return and Endogenous Capitalization

The concept of endogene calculated profitablity offered by honourable Dr. Lutz Kruschwitz (1995, 2000) in his relevant book "*Investitionsrechnung* (6th ed.)" contains serious mistake what remains also in the latest 2000 edition „*Investitionsrechnung* (8th ed.)".

Let us start with comparing following two tables (Table 4 and 5).

Obviously, the Tables 4 and 5 differ substantially. The principal mistake is the value 0,11 for A and value 0,09 for I. The proper endogenous capitalizations are 0 by Complementary Slackness Conditions (Ravindran, Phillips and Solberg, 1987). These mistakes signalize that joint investing and financing program requires further detail investigation.

The demonstration of idea of total information analysis of problem (I•F) with help of simplex method in pivot transformation form comes next.

Total Information Analysis of Joint Investing and Financing Intertemporal Problem (I•F) with Help of Eduard Stiefel[1] Simplex Method

Let us utilize the method of Eduard Stiefel (or Tucker-Balinski pivot transformation). This method is also known as Jordan Elimination method in Linear Programming (Stiefel, 1960; Balinski and Tucker, 1969; Jaunzems, 1981, 1990, 1993).

Table 6 contains the initial system of equations for problem (I•F). Table 7

Table 4

The endogenous capitalizations calculated by Dr. Lutz Kruschwitz

A	B	C	D	E	F	G	H	I	J	K	L	M	N
INV_1	INV_2	INV_3	INV_4	INV_5	INV_6	INV_7	INV_8	FIN_1	FIN_2	FIN_3	FIN_4	FIN_5	FIN_6
0.11	44.36	23.36	–11.66	–2.40	0.00	–3.16	–1.72	**0.09**	–6.40	–1.28	–3.48	0.00	–1.20

Source: Kruschwitz, 2000 (p. 191).

Table 5

The true endogenous capitalizations

A	B	C	D	E	F	G	H	I	J	K	L	M	N
INV_1	INV_2	INV_3	INV_4	INV_5	INV_6	INV_7	INV_8	FIN_1	FIN_2	FIN_3	FIN_4	FIN_5	FIN_6
0	44.42	23.48	–11.50	–2.38	0	–3.16	–1.72	**0**	–6.42	–1.30	–3.47	0	–1.20

Table 6

Initial simplex table for problem (I•F)

Initial	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	y_1	y_2	y_3	y_4	y_5	y_6	*niveau*	1
v_1	0	–800	–700	–300	–100	0	0	0	1000	600	100	0	0	0	0	500
v_2	–500	80	500	700	106	–100	0	0	–80	0	–110	100	0	0	–1	0
v_3	–900	160	300	350	0	106	–100	0	–388	0	0	–110	100	0	–1,05	0
v_4	1250	320	–200	170	0	0	106	–100	–388	0	0	0	–110	100	–1,1	0
v_5	350	520	220	–1090	0	0	0	106	–388	–832	0	0	0	–110	–1,15	–500
u_1	–1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
u_2	0	–1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
u_3	0	0	–1	0	0	0	0	0	0	0	0	0	0	0	0	1
u_4	0	0	0	–1	0	0	0	0	0	0	0	0	0	0	0	2
w_2	0	0	0	0	0	0	0	0	0	–1	0	0	0	0	0	1
Objective	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

Table 7

Ending simplex table for problem (I•F)

Finish	v_1	v_2	v_3	v_4	x_5	x_4	x_7	x_8	u_2	y_2	y_3	y_4	v_5	y_6	u_3	1
x_1	0.0002	–0.0009	–0.0008	–0.0007	0.1086	3.4362	–0.0029	–0.3045	0.5576	–1.7060	–0.1121	–0.0032	0.0022	0.3132	–0.1692	0.6117
x_2	0	0	0	0	0	0	0	0	–1	0	0	0	0	0	0	1
x_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–1	1
x_6	0.0011	–0.0031	0.0065	0.0059	0.4435	–10.3884	0.0236	1.5271	–2.8226	–8.0321	–0.4560	1.0259	–0.0088	–1.5625	–3.5276	1.3667
y_5	0.0013	–0.0071	–0.0067	–0.0152	0.8886	39.5673	0.9392	–4.3639	6.3749	21.5213	–0.9170	–0.0268	0.0268	4.4713	2.4307	3.9400
u_1	–0.0002	0.0009	0.0008	0.0007	–0.1086	–3.4362	0.0029	0.3045	–0.5576	–1.7060	0.1121	0.0032	–0.0022	–0.3132	0.1692	0.3883
y_1	0.0010	0	0	0	0.1	0.3	0	0	–0.8	–0.6	–0.1	0	0	0	–0.7	1
niveau	–0.2825	–0.2601	–0.2454	–0.2231	–0.6735	–3.2481	–0.8924	–0.4857	–12.5487	–1.8134	–0.3670	–0.9816	–0.2059	–0.3378	–6.6316	57.4745
u_4	0	0	0	0	0	–1	0	0	0	0	0	0	0	0	0	2

Table 8

The duality equations for problem (I•F)

0	–800	–700	–300	–100	0	0	0	1000	600	100	0	0	0	0	0.2825
–500	80	500	700	106	–100	0	0	–80	0	–110	100	0	0	–1	0.2601
–900	160	300	350	0	106	–100	0	–388	0	0	–110	100	0	–1.05	0.2454
1250	320	–200	170	0	0	106	–100	–388	0	0	0	–110	100	–1.1	0.2231
350	520	220	–1090	0	0	0	106	–388	–832	0	0	0	–110	–1.15	0.2059
0.0000	12.5487	6.6316	–3.2481	–0.6735	0.0000	–0.8924	–0.4857	0.0000	–1.8134	–0.3670	–0.9816	0.0000	–0.3378	–1.0000	

Table 9

The duality equations for problem (I•F) in the transformed form

0	–800	–700	–300	–100	0	0	0	1000	600	100	0	0	0	0	1.0000
–500	80	500	700	106	–100	0	0	–80	0	–110	100	0	0	–1	0.9209
–900	160	300	350	0	106	–100	0	–388	0	0	–110	100	0	–1.05	0.8688
1250	320	–200	170	0	0	106	–100	–388	0	0	0	–110	100	–1.1	0.7898
350	520	220	–1090	0	0	0	106	–388	–832	0	0	0	–110	–1.15	0.7289
0	44.4236	23.4764	–11.4985	–2.3843	0	–3.1592	–1.7195	0.0000	–6.4195	–1.2993	–3.4751	0	–1.1960	–3.5401	

Table 10

The duality equations in the transformed form in order to show the endogenous rates of return

0	–800	–700	–300	–100	0	0	0	1000	600	100	0	0	0	0	1
–500	80	500	700	106	–100	0	0	–80	0	–110	100	0	0	–1	$(1+0.08589)^{-1}$
–900	160	300	350	0	106	–100	0	–388	0	0	–110	100	0	–1.05	$(1+0.07288)^{-2}$
1250	320	–200	170	0	0	106	–100	–388	0	0	0	–110	100	–1.1	$(1+0.08185)^{-3}$
350	520	220	–1090	0	0	0	106	–388	–832	0	0	0	–110	–1.15	$(1+0.08229)^{-4}$
0	44.4236	23.4764	–11.4985	–2.3843	0	–3.1592	–1.7195	0.0000	–6.4195	–1.2993	–3.4751	0	–1.1960	3.5401	

contains the ending system of equations for problem (I•F).

Important remark. Let us stress that initial simplex table means only system of linear equations and nothing else. For example, $v_1 = 0\ x_1 - 800\ x_2 - 700\ x_3 - 300\ x_4 - 100\ x_5 + 1000\ y_1 + 600\ y_2 + 100\ y_3 + 500$.

Analogicaly, the ending simplex table also means only the system of linear equations algebraically equivalent to the initial system and nothing else. For example, *niveau* = $-0.2825\ v_1 - 0.2601\ v_2 - 0.2454\ v_3 - ... + 57.4745$.

Each number in Tables 6 and 7 can be pithy interpreted in the context of joint investing and financing problem (I•F).

Obviously, the *niveau* takes the maximal value 57.4745 then and only then if $v_1 = 0$, $v_2 = 0$, $v_3 = 0$, $v_4 = 0$, $x_4 = 0$, $x_5 = 0$, $x_7 = 0$, $x_8 = 0$, $u_2 = 0$, $u_3 = 0$, $y_2 = 0$, $y_3 = 0$, $y_4 = 0$, $y_5 = 0$, $y_6 = 0$.

It is necessary to stress that the shadow prices 0.2825; 0.2601; 0.2454; 0.2231; 0.2059 are crucially important because these indicators measure the marginal contribution of intertemporal liquidity (MCIL) in the objective – maximum *niveau*.

For example, −0.2454 is the coefficient by v_3 in final expression of *niveau*. The value $v_3 = -1$ means that Mr. Jones invests in program at moment t = 2 one additional external dollar. As a result, the maximum *niveau* will increase by 0.2454 dollars.

Shadow prices and reduced costs are used by Dr. Lutz Kruschwitz for endogenous capitalizations and endogenous rates of return calculation. We also will do that.

Consider the solution of the dual optimization problem. According to the concept of duality, the system of equations expressed in the Table 8 holds.

Remark. In the Table 8, equations are inscribed vertically. For example, equation corresponds to the second column:

$$-800 \cdot 0.2825 + 80 \cdot 0.2601 + 160 \cdot 0.2454 + 320 \cdot 0.2231 + 520 \cdot 0.2059 = 12.5487.$$

The fourth column means equation:

$$-300 \cdot 0.2825 + 700 \cdot 0.2601 + 350 \cdot 0.2454 + 170 \cdot 0.2231 - 1090 \cdot 0.2059 = -3.2481.$$

Let us stress that with help of direct and dual simplex systems is possible to make the fundamental sensitivity analysis of problem (I•F).

It is easy to obtain the endogenous capitalizations and endogenous rates of return with help of duality equations. Dividing by 0.2825, the dual system can be rewritten in the form of Table 9. Now we obtain numbers contained in Table 5 which sufficiently differ from numbers in Table 4.

In Table 9, duality equations in a transformed form are presented. Right-side column shows the calculated endogenous discount coefficients. Table 9 lets us to interpret endogenous rates of return and endogenous capitalization.

Table 10 is almost the same as Table 9. Only difference is that the numbers in the last column are transformed in form to obtain endogenous rates of return.

We have got $i_{0,1} = 8.589\%$; $i_{0,2} = 7.288\%$; $i_{0,3} = 8.185\%$; $i_{0,4} = 8.229\%$.

For example, the first column contains the equality:

$$0 \cdot 1 - 500 \cdot (1 + 0.08589)^{-1} - 900 \cdot (1 + 0.07288)^{-2} + 1250 \cdot (1 + 0.08185)^{-3} + 350 \cdot (1 + 0.08229)^{-4} = 0.$$

The second column contains the equality:

$$-800 \cdot 1 + 80 \cdot (1 + 0.08589)^{-1} + 160 \cdot (1 + 0.07288)^{-2} + 320 \cdot (1 + 0.08185)^{-3} + 520 \cdot (1 + 0.08229)^{-4} = 44.4236.$$

The fourth column contains the equality:

$-300 \cdot 1 + 700 \cdot (1+0{,}08589)^{-1} + 350 \cdot (1+0{,}07288)^{-2} + 170 \cdot (1+0{,}08185)^{-3} - 1090 \cdot (1+0{,}08229)^{-4} = -11{,}4985.$

Thus, the Table 10 allows us to interpret the endogenous capitalizations and endogenous rates of return. The bottom row's numbers in the Table 10 (of course, except –3.5401) can be interpreted as endogenous capitalizations of problem's (I•F) cash flows calculated by using endogenous rates of return.

Conclusions

I confirm that the contents of endogenous rates of return and endogenous capitalizations introduced by Dr. Lutz Kruschwitz can be explained only by means of the Stiefel (1960) systems of equation stated in Tables 7, 8, 9, 10 and theorem of duality. These systems contain in the explicit form absolutely the same information as initial optimization problem (I•F) in implicit form.

Let us assume that joint investing and financing problem (I•F) is regular in a such sense that Complementary Slackness Conditions of optimality (Ravindran et al., 1987) fulfil in the strong "then and only then" form. For such problem (I•F) optimality conditions $x^*_i \cdot v^*_i = 0$, $y^*_j \cdot u^*_j = 0$ mean that $x^*_i > 0 \Leftrightarrow v^*_i = 0; y^*_j > 0 \Rightarrow u^*_j = 0.$

From the theory of duality in case of regular problem (I•F), follows the theorem what I have never met in scientific literature.

Theorem about joint investing and financing program's endogenous capitalization. Let us assume that the linear program by means of which endogenous indicators are calculated is regular, then the following statements are valid:

1. Endogenous capitalization of investing or financing cash flow is zero then and only then, if the optimal financial program utilizes this cash flow at intermediate level i. e. limitedness condition is not active (or utilizing of this cash flow is not limited at all).
2. Endogenous capitalization of investing or financing cash flow is positive then and only then, if the optimal financial program utilizes this cash flow at limit i. e. limitedness condition is active.
3. Endogenous capitalization of investing or financing cash flow is negative then and only then, if the optimal financial program does not utilize this cash flow.

Notes

[1] Eduard Stiefel (1909–1978). Professor of Mathematics at ETH Zürich (*Eidgenössische Technische Hochschule Zürich*).

References

1. Balinski, M. L., & Tucker, A. W. (1969). Duality theory of linear programs: A constructive approach with applications. *SIAM Review*, 11(3), 347–377.
2. Bodie, Z., Cane, A., Marcus, A. J., Perrakis, S., & Ryan, P. J. (1999). *Investments* (3rd ed.). McGraw-Hill: Ryerson.
3. Bodie, Z., & Merton R. C. (2000). *Finance*. Prentice Hall: Pearson Education Company.
4. Jaunzems, A. (1981). *Lineārās algebras un lineārās analīzes pamati*. Rīga: Zvaigzne.
5. Jaunzems, A. (1993). *Математика для экономических наук. Общий курс*. Латвийский Университет.

6. Jaunzems, A. (1990). *Линейное оптимальное планирование*. Латвийский Университет.
7. Jaunzems, A. (2001). Investīciju un finansēšanas projekta endogēnie raksturotājlielumi In Starptautiska zinātniska konferences *Ekonomisko un sociālo attiecību transformācija: procesi, tendences, rezultāti* rakstu krājums (p. 144–147). Rīga, Biznesa Augstskola Turība.
8. Jaunzems, A. (2008). Investīciju un finansēšanas programmas analīze. In *Latvijas Universitātes raksti. Sējums 737. Ekonomika,* VII (pp. 181–195). Rīga: Latvijas Universitāte.
9. Jaunzems, A. (2009). Finanšu matemātika. Trešais izdevums, papildināts un pārstrādāts. Ventspils Augstskola: Drukātava.
10. Jaunzems, A. (2012). Immunized quasi-arbitrage. *Journal of Business Management*, 47–56.
11. Kruschwitz, L. (1995). *Investitionsrechnung. 6. Auflage.* Walter de Gruyter.
12. Kruschwitz, L. (2000). *Investitionsrechnung. 8., neu bearbeitete Auflage.* R. Oldenbourg Wissenschaftsverlag.
13. Kruschwitz, L. (2001). *Инвестиционные расчеты*. Питер.
14. Ravindran, A., Phillips, D. T., & Solberg, J. J. (1987). *Operations research. Principles and practice* (2nd ed.). John Wiley & Sons.
15. Seo, K. K. (2000). *Управленческая экономика*. Москва: Инфра-М.
16. Sneps-Sneppe, M. (2017a). How to measure the effectiveness of investments: Criticism of the arguments of Pierre Masse. *International Journal of Open Information Tehnologies*, 5(8), 7–11.
17. Sneps-Sneppe, M. (2017b). On the internal rate of return of IRR and the priority of investments. *International Journal of Open Information Tehnologies*, 5(9), 39–43.
18. Sneps-Sneppe, M. (2017c). The problem of investment during the history of economic science. *International Journal of Open Information Tehnologies*, 5(9), 34–38.
19. Stiefel, E. (1960). Note on Jordan elimination, linear programming and Tchebycheff approximation. *Numerische Mathematik*, 2, 1–17.
20. Van Horne, J. C., & Vachowiz Jr., J. M. (2001). *Fundamentals of financial management* (11th ed.). Prentice Hall: Pearson Education Company.
21. Van Horne, J. C., & Vachowiz Jr., J. M. (2003). *Fundamentals of financial management* (10th ed.). Moscow: Williams.

The paper submitted July 11, 2017
Prepared for publication June 1, 2018

Andrejs JAUNZEMS

ENDOGENINIAI JUNGTINĖS TARPLAIKINĖS FINANSAVIMO IR INVESTAVIMO PROGRAMOS VEIKSNIAI

Santrauka

Kapitalo sudarymo metodų tyrimai, minimi visuose populiariuose finansų valdymo vadovėliuose, rodo, kad vadybininkai, kaip įprasta, naudoja tokius populiarius pinigų srautų rodiklius, kaip vidinės grąžos norma, grynoji dabartinė vertė ir pelningumo indeksas, suteikdami jiems skirtingus sprendimų priėmimo svorius. Kyla klausimas – kuriam rodikliui privalo būti skiriama pirmenybė? Autorius teigia, kad problema, susijusi su pirmenybės nustatymu tarp grynosios dabartinės vertės ar vidinės grąžos normos, neegzistuoja, nes nėra esminio skirtumo tarp šių vartojamų sąvokų. Svarbiausia pinigų srautų analizės priemonė, kaip visos palūkanų normos funkcija, yra grynoji dabartinė vertė. Pagrindinė turto vertė – investuotojams naudingas rodiklis, tačiau vidinė grąžos norma (arba pelningumas) yra dažniausiai naudojamas rodiklis realių verslo projektų pelningumui vertinti. Empiriniai tyrimai patvirtina šį teiginį. Tačiau viso to nepakanka, siekiant analizuoti vieną ar kitą investiciją ar finansinį projektą. Versle viena svarbiausių problemų yra įmonės aplinkos PESTILB (angl. *political, economical, social, technological, international, legal, bio-environmental*) analizė ir ypač plati investavimo bei finansavimo analizė. Jungtinių tarplaikinių investavimo ir finansavimo projektų pelnas ir nuostolis turi būti ištirti kompleksiškai. Tuo pačiu metu atrankos kriterijai tarp potencialių investavimo ir finansavimo projektų, siekiant sukurti priimtiną ilgo laikotarpio jungtinį investavimo ir finansavimo portfelį, tikriausiai yra labiausiai ginčytini finansų valdymo klausimai. Šis straipsnis skirtas jungtinių

investavimo ir finansavimo projektų portfeliui formuoti, kai nustatomi pinigų srautai. Pagrindinė modelio prielaida: investuotojas gali formuoti portfelį, kurio pinigų srautai yra tiesinė potencialių, galimų atskirų investicijų ir atskirų finansinių projektų pinigų srautų kombinacija. Įgyvendinus finansinę programą, kuri yra grindžiama išoriniais pinigų srautais, investuotojas gauna endogeninius pinigų srautus. Tyrimo objektas yra prieinami endogeniniai pinigų srautai, kurie nustatyti priklausomai nuo egzogeninio investavimo vektoriaus. Tokiu būdu investuotojas maksimizuoja naudą: investuotojas pasirenka geriausią portfelį, kurį gali sau leisti įsigyti. Dėl šios priežasties finansinės programos pasirinkimas gali būti interpretuojamas kaip tarplaikiniai pinigų srautų mainai, susiejant šį veiksmą su vartotojo elgsenos teorija. Šiuo tyrimu siekiama apibūdinti abipusę tarpusavio priklausomybę tarp skirtingų finansinių rodiklių. Toks tarpusavio priklausomumas suteikia galimybę gauti finansinių rodiklių interpretacijas. Vertinimas buvo atliktas taikant *Eduard Stiefel* kompleksinį metodą (taip pat žinomas kaip *Tucker-Balinski* sukimosi tranformavimo metodas). Šis straipsnis skirtas tolesnėms diskusijoms su didžiai gerbiamu dr. Kruschwitz (1995, 2000) dėl apskaičiuotos endogeninės grąžos normos (vok. *Endogene Kalkulationszinsfüße*) ir endogeninės kapitalizacijos (vok. *Endogene Kapitalwert*) reikšmių, kurios aptariamos jo knygose „Investitionsrechnung“ (6 ir 8 leidimas). Straipsnyje siekiama įrodyti, kad rimta pelningumo skaičiavimo, pasiūlyto gerbiamo dr. Kruschwitz, klaida pastebima ir naujausiuose šios garsios knygos leidimuose.

Autorius patvirtina, kad endogeninės grąžos normos ir endogeninės kapitalizacijos esmė, pristatyta dr. Lutz Kruschwitz, gali būti paaiškinta tik naudojant *Eduard Stiefel's* lygčių sistemos ir dualumo teoremos reikšmes. Tiek tiesiogine, tiek netiesiogine optimizavimo problemos forma pateikiama visiškai ta pati informacija.

Galima teigti, kad ši teorema apie jungtinių investavimo ir finansavimo projektų endogeninę kapitalizaciją yra nauja, niekada anksčiau nesutikta mokslinėje literatūroje. Teorema: daroma prielaida, kad portfelio kapitalo lygis maksimaliai padidintas, o tiesinės programos, pagal kurią apskaičiuojami endogeniniai rodikliai, papildomos atsparumo sąlygos yra tenkinamos griežta „tada ir tik tada“ forma. Teoremos teiginiai ir sąlygos: 1) endogeniniai investicijų kapitalizavimo ar finansavimo pinigų srautai yra lygūs nuliui tada ir tik tada, jei optimali finansavimo programa naudoja šiuos pinigų srautus tarpiniu lygmeniu, t. y. ribotumo sąlyga nėra tenkinama (arba šių pinigų srautų naudojimas yra visai neapribojamas); 2) endogeniniai investicijų kapitalizavimo ar finansavimo pinigų srautai yra teigiami tada ir tik tada, jei optimali finansavimo programa naudoja šiuos pinigų srautus neviršijant tam tikrų ribų (t. y. tenkinama apribojimo sąlyga); 3) endogeniniai investicijų kapitalizavimo ar finansavimo pinigų srautai yra neigiami tada ir tik tada, jei optimali finansavimo programa nenaudoja šių pinigų srautų.