THEORETICAL CALCULUS MODEL FOR THE FINITE LENGTH RAYLEIGH STEP BEARINGS LUBRICATED WITH GREASES

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Abstract: The present paper proposes a calculus methodology for the finite length Rayleigh step bearings (3D case), lubricated with greases. In the case of a Rayleigh step bearing, the main problem involved is the presence of the stagnant core of grease into the region of the step. The shape of the stagnant core is obtained by finite element method, applied for solving the Navier-Stokes equations. Once established the dimensions of the stagnant core, the characteristic parameters of the bearing (pressure distribution, load, flow capacity and apparent friction coefficient) will be numerically determined, using the finite differences method.

Keywords: Rayleigh step bearing, Grease, Finite element method, Finite differences method

1. INTRODUCTION

Considering the design process of the machine elements, it can be observed that the Rayleigh step bearing with finite length (3D case), with ratio B/L < 2, is very appropriate for modeling the slider bearings [10]. Comparing with the infinite large Rayleigh step bearing (B/L → ∞), in this 3D case there are some difficulties due to the assumption of the bi-directional grease flow along the coordinate axis Ox and Oz. Analysing references from literature, the infinite large modelisation continues to be used for its simplicity from the analytical point of view, with the advantage of some qualitative information, available also for the finite bearing [1-5].

Taking into account the conclusions concerning the grease lubricated Rayleigh step bearing [8-9], with B/L → ∞, in this paper an original calculus method is presented for 3D case.

2. THEORETICAL MODEL AND FUNDAMENTAL HYPOTHESIS

Figure 1 presents the geometry of the finite length Rayleigh step bearing (3D case) lubricated with grease, including the shape of the stagnant regions.

Analysing the proposed model from Figure 1, it can be observed the similarity with the finite length Rayleigh step bearing lubricated with oil. The main difference is the presence of the stagnant core of grease, which reduce the film thickness in the inlet region.

The main assumptions used in this case are:

- The rheological three-dimensional model for the behavior of the grease is [6]:

\[
\bar{T} = \begin{cases} 
2m(-4D_{II})^{-1} + \frac{\tau_0}{\sqrt{-D_{II}}} & \text{for } \bar{D} \neq 0 \\
\sqrt{-T_{II}} \leq \tau_0 & \text{for } \bar{D} = 0 
\end{cases}
\]

where: \(\bar{T}\), \(\bar{D}\) - tensors of shear and strain rate; \(T_{II} : D_{II}\) - second invariant for the tensor of shear and strain rate; \(m\) - flow index; \(n\) - consistency index; \(\tau_0\) - yield stress.

- The flow of the grease is considered isothermal.

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Before the step, in the superior region of the bearing (zone I: \( x \in [0; L_1] \), \( y \in [h'_1; h_1] \) and \( z \in [-b_z/2; b_z/2] \)), the entire lubricant film becomes a stagnant core, due to the viscoplastic comportment of the grease. The shape and the dimensions of the stagnant region have been determined from the theoretical point of view (see Figure 2), using the finite element method for the integration of the Navier-Stokes equations. This profile was also obtained from experiments, using the visualisation of the flow by reflecting tracer method [7].

\[ x \in [0; L_1], \quad y \in [h'_1; h_1] \quad \text{and} \quad z \in [-b_z/2; b_z/2] \]

\[ \text{stagnant core (zone I)} \]

\[ \text{Figure 1. Geometry of finite length Rayleigh step bearing lubricated with grease.} \]
a) B/L = 0.5

b) B/L = 1

c) B/L = 2

Figure 2. The shape and dimensions for the stagnant region (B/L = 1, $A = r_0 h^m / mU^n = 10$ and $h_1/h_2 = 2.5$).

- If the length of the studied region is significant bigger than the film thickness, the stagnant core of the grease becomes in fact a parabolic cylinder, with the following characteristics parameters for the inferior frontier: thickness $h_1^*$ and width $b_1^*$.

- In the inferior region, before (the zone II: $x \in [0;L_1]$, $y \in [0;h_1^*]$ and $z \in [-B/2;B/2]$) and after (the zone III: $x \in [L_1;L]$, $y \in [0;h_2^*]$ and $z \in [-B/2;B/2]$) the step, the fluid behaves as a Newtonian medium. Therefore, in this particular case, for both regions, the flow of lubricant is equivalent with a Couette - Poiseuille flow.

3. CHARACTERISTIC PARAMETERS FOR THE RAYLEIGH STEP BEARING

The calculus of the characteristics parameters for this bearing suppose the integration of the bi-dimensional Reynolds equation for the zones II and III, where the lubricant has a viscous behaviour:

$$\frac{\partial}{\partial x} \left( h^{n+2} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( h^{n+2} \frac{\partial p}{\partial z} \right) = 6mU^n \frac{\partial h}{\partial x}$$  \hspace{1cm} (2)

where: $U$ - velocity of the inferior surface; $m$ - flow index; $n$ - consistency index; $p$ - pressure; $h$ - film thickness; $x$, $z$ - co-ordinates in a Cartesian system.

The equation has been solved with the finite difference method, the pressure distribution, load capacity, flow capacity and apparent friction coefficient being determined:
• The pressure distribution is presented comparatively in Figure 3, in the case of Newtonian and non-Newtonian fluid.

\[ A = \tau_0 h^n / mU^n \]

a) \( A = \tau_0 h^n / mU^n = 0 \)

b) \( A = \tau_0 h^n / mU^n = 10 \)

**Figure 3.** The pressure distribution for the newtonian \((A = 0)\) and non-newtonian \((A = 10)\) fluid \((B/L = 1; h_1/h_2 = 2.5)\).

• The flow capacity (Figure 4):
  - the inlet flow capacity:
  \[
  Q_{zs} = \int_0^h \int_0^L \left[ -\frac{1}{2m} \frac{\partial p}{\partial x} y(h - y) + \left( 1 - \frac{y}{h} \right) U \right] dydz
  \]
  \((3)\)
  - the lateral flow capacity:
  \[
  Q_{zs} = \int_0^h \int_0^L \left[ -\frac{1}{2m} \frac{\partial p}{\partial x} y(h - y) \right] dydz
  \]
  \((4)\)
  - the outlet flow capacity:
  \[
  Q_{ex} = Q_{zs} - Q_{zs}
  \]
  \((5)\)
The variation of the non-dimensional flow capacity for the Rayleigh step bearing lubricated with grease (B/L = 1).

- The non-dimensional load capacity (Figure 5):

Figure 4.
\[ F = \int_{0}^{1} \int_{0}^{1} \bar{p} \, d\xi \, d\zeta \]  

(6)

where:  \( F = F h_2^2 / mUBL^2 \) - non-dimensional load;  \( \bar{p} = ph_2^2 / mUL \) - non-dimensional pressure.

\[ f_0 = \frac{\bar{F}_f}{\bar{F}} \]  

(7)

where:  \( \bar{F}_f = F_f h_1 / mUBL \) - non-dimensional friction force.

**Figure 5.** The variation of the non-dimensional load capacity for the Rayleigh step bearing lubricated with grease (B/L = 1).

- The apparent friction coefficient (Figure 6):

**Figure 6.** The variation of the apparent friction coefficient for the Rayleigh step bearing lubricated with grease (B/L = 1).

4. CONCLUSIONS

1. In the case of a Rayleigh step bearing lubricated with grease, the extremes values of the characteristic parameters (load capacity, flow and apparent friction coefficient) are obtained for bigger values of the ratio \( h_1/h_2 \) than in the case of a bearing lubricated with oil.

2. For values of ratio \( h_1/h_2 \) bigger then 3, the characteristic parameters of the bearing lubricated with grease are almost constant, for all the values of the ratio \( h_1/h_2 \). The importance of this conclusion
consists in demonstrating that the grease lubricated bearing is not so sensible as a bearing lubricated with oil bearing, regarding the dimension of the step \( h_1 - h_2 \).

3. Looking at all the values of the characteristic parameters of the bearings, it can be observed that the optimum ratio \( L = \frac{L_1}{L} \) is identically \( (L_1 = \frac{L_1}{L} = 0.7) \) for both cases: grease and oil lubricated bearings.

REFERENCES